Solution #8 (20 points) Mat 1371 Mayer Alvo

Chapter 16

8. Carnival. [5 points]

a)

Net winnings	\$95	\$90	\$85	\$80	−\$20
Number of darts	1	2	3	4	5
P(amount won)	0.1	0.9(.1)	(0.81)(.1)	(0.729)(.1)	0.6561

b) E(number of darts) = 1(0.1) + 2(0.09) + 3(0.081) + 4(0.0729) + 4(0.6561) ≈ 3.44
darts

c) E(winnings) = \$95(0.1) + \$90(0.09) + \$85(0.081) + \$80(0.0729) - \$20(0.6561) ≈
\$17.20

22. Day trading again. [5 points]

a) E(stock option) · 1000(0.20) · 0(0.30) · 200(0.50) · \$300

The trader should buy the stock option. Its expected value is \$300, and he only has to pay \$200 for it.

b) E(gain) · 800(0.20) · (· 200)(0.30) · 0(0.50) · \$100

The trader expects to gain \$100. Notice that this is the same result as subtracting the \$200 price of the stock option from the \$300 expected value.

c) Var (gain) = $(800 \cdot 100)^2$ (0.20) = $(-200 - 100)^2$ (0.30) = 130,000

SD(gain) = \$360.56

Notice that the standard deviation of the trader's gain is the same as the standard deviation in value of the stock option

30. Random variables. [5 points]

a)

$E(2Y \cdot 20) = 2(E(Y)) - 20 = 44$

$SD(2Y + 20) = 2(SD(Y)) = 2(3) = 6$

b) $E(3X) = 3(E(X)) = 3(80) = 240$

$SD(3X) = 3(SD(X)) = 3(12) = 36$

c) $E(0.25X + Y) = 0.25(E(X)) + E(Y) = 0.25(80) + 12 = 32$

$SD(0.25X + Y) = \text{sqrt}(\ 0.25^2 Var(X) + Var(Y) = 0.25^2(12^2) + 3^2 \cdot\ ) = 4.24$

d) $E(X - 5Y) = E(X) - 5(E(Y)) = 80 - 5(12) = 20$

$SD(X \cdot\ 5Y) = \text{sqrt}[\ Var(X) + 5^2 Var(Y)] = 19.21$

e) $E(X_1 + X_2 + X_3) = E(X) + E(X) + E(X) = 80 + 80 + 80 = 240$

$SD(X_1 + X_2 + X_3) = \text{sqrt}[\ Var(X_1) + Var(X_2) + Var(X_3)] = 20.78$

44. Bikes. [5 points]

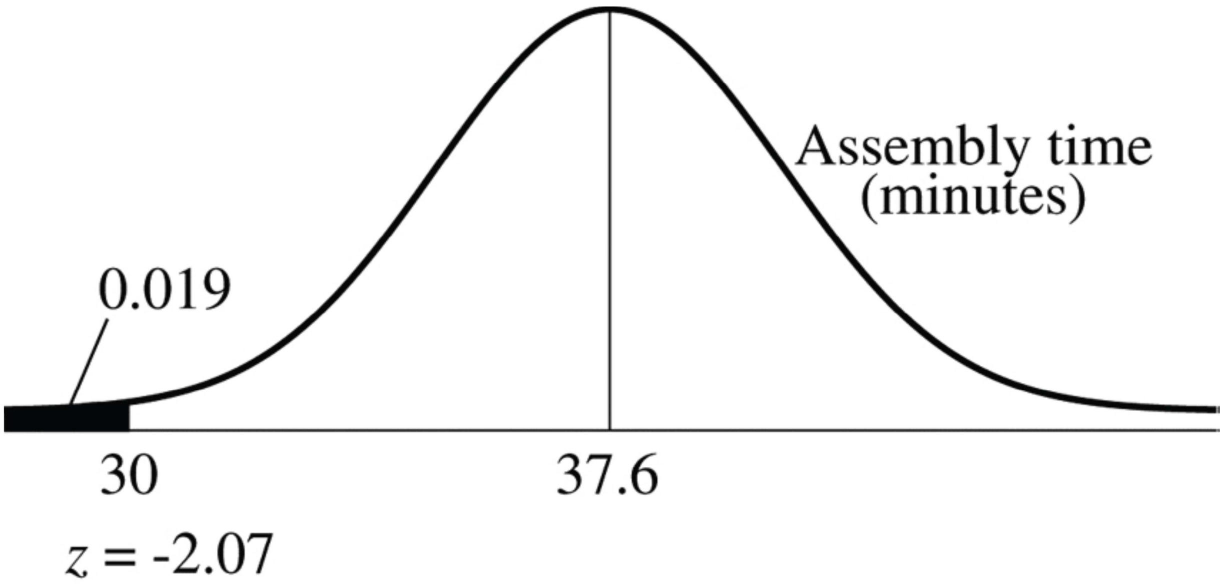
a) E(unpack + assembly + tuning) = E(unpack) + E(assembly) + E(tuning)

= 3.5 + 21.8 + 12.3 = 37.6 minutes

SD(unpack + assembly + tuning) =sqrt [Var(unpack) +Var(assembly) +Var(tuning)]

= 3.7 minutes

b) The bike is not likely to be ready on time. According to the Normal model, the probability that an assembly is completed in under 30 minutes is about 0.019.



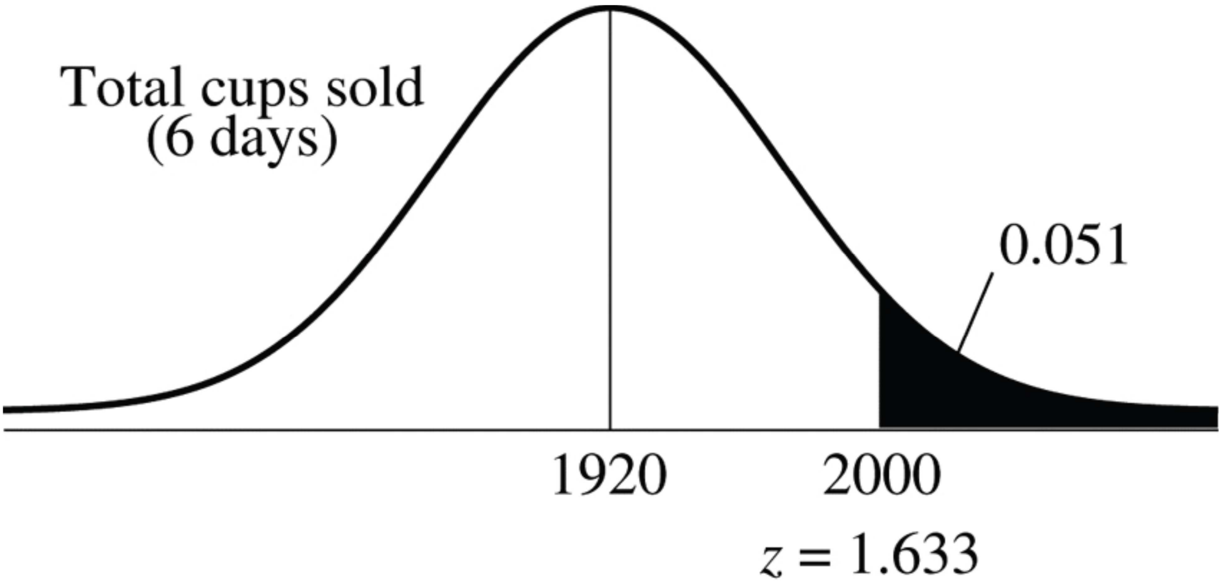
47. Coffee and doughnuts. [5 points]

a) E(cups sold in 6 days) = 6(E(cups sold in 1 day)) = 6(320) = 1920 cups

SD(cups sold in 6 days) = sqrt [6(Var (cups sold in 1 day)]= 48.99 cups

The distribution of total coffee sales for 6 days has distribution N(1920, 48.99).

According to the Normal model, the probability that he will sell more than 2000 cups of coffee in a week is approximately 0.051.



b) Let C = the number of cups of coffee sold. Let D = the number of doughnuts sold.

$E(50C + 40D) = 0.50(E(C)) + 0.40(E(D)) = 0.50(320) + 0.40(150) = \$220$

$SD(0.50C + 0.40D)$ =sqrt [$0.50^2\,(Var(C)) + 0.40^2\,(Var(D))$] = \$11.09

The day' s profit can be modeled by N(220, 11.09). A day' s profit of \$300 is over 7 standard deviations above the mean. This is extremely unlikely. It would not be reasonable for the shop owner to expect the day' s profit to exceed \$300.

c) Consider the difference $D - 0.5C$. When this difference is greater than zero, the number of doughnuts sold is greater than half the number of cups of coffee sold.

$E(D - 0.5C) = (E(D)) - 0.5(E(C)) = 150 - 0.5(320) = -\$10$

$SD(D - 0.5C)$ =sqrt [$(Var(D)) - 0.5(Var(C))$] = \$15.62

The difference $D\cdot$ $0.5C$ can be modeled by $N(-10, 15.62)$.

According to the Normal model, the probability that the shop owner will sell a doughnut to more than half of the coffee customers is approximately 0.26.